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A New Odd Exponential-Weibull Distribution with Applications to Survival Dataset

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ABSTRACT

Survival data are very common in clinical, chemical, and agronomic assays, among others. However, in practice, experiments are conducted so that all sample units are evaluated at the same time. These data are referred to as grouped survival data, which are a particular case of interval censoring and are characterized by an excessive number of ties. The study examines some of the existing parametric distributional models in accommodating various datasets and develops an extension termed the new odd exponential-Weibull distribution with applications to survival datasets. The research methodology used PDF and CDF plots of the Odd Exponential-Weibull Distribution keeping one parameter constant and varying others. The results presented in Table 1 reflect the comparative analysis of some statistical models fitted to the AAML dataset, with an emphasis on their goodness-of-fit measures. The table provides parameters and their estimates for each model, along with standard errors and various goodness-of-fit criteria such as Log-Likelihood (LL), Akaike Information Criterion (AIC), Consistent Akaike Information Criterion (CAIC), Bayesian Information Criterion (BIC), and Hannan-Quinn Information Criterion (HQIC). The log-likelihood (LL) values indicate the likelihood of the data given the model parameters. Among the models, the OEW model has the highest LL (-391.3732), suggesting it provides the best fit to the data by maximizing the likelihood. The AIC values assess the trade-off between the goodness of fit and the complexity of the model, where lower values indicate a better model. The research concluded by building upon privious work, this new distribution enhanced flexibility across several datasets. Thus, the study introduced the new distribution and evaluated its performance using lifetime and survival datasets. In addition, the research established the potential of the developed new distribution and its variants as promising alternatives in modelling positive data.

Keywords: survival; applications; new; odd; exponential; Weibull; distribution

INTRODUCTION

In probability theory and statistics, the exponential distribution also known as a negative exponential distribution is a continuous probability distribution that often concerns the amount of time until some specific event happens. It is a process in which events happen continuously and independently at a constant average rate. For example, the amount of money spent by the customer on one trip to the supermarket follows an exponential distribution. The exponential distribution is a probability distribution of time between events in the Poisson point process. The exponential distribution is considered as a special case of the Gamma distribution. Also, the exponential distribution is a continuous analogue of the geometric distribution, and it has the key property of being memoryless. While the exponential distribution models the time until the next event occurs, the related gamma distribution models the time until the k^{th} event occurs, where k is the shape parameter. Conversely, the Poisson distribution models the count of events within a fixed amount of time. The exponential distribution is a continuous probability distribution that models the variables in which small values occur more frequently than higher values. Statisticians use the exponential distribution to model the amount of change in people's pockets, the length of phone calls, and sales totals for customers. In all these cases, small values are more likely than larger values. Researchers in the medical sciences analyze the lifetime of dental and medical implants and are often in search of techniques to model certain risk factors in patients with rare or particular diseases toward survival (Edward *et al.,* 2021).

Daniel *et al.,* (2022) propose a new probability distribution called Exponential-Exponential distribution, provide a comprehensive study of its theory and derive appropriate expressions for its statistical properties. The method of maximum likelihood was employed to estimate its parameters.

Magnus and Magnus (2019) proposed that the method of maximum likelihood estimation (MLE) has great intuitive appeal and generates estimators with desirable asymptotic properties. The estimators are obtained by maximization of the likelihood function, and the asymptotic precision of the estimators is measured by the inverse of the information matrix. Thus, both the first and second differential of the likelihood function need to be found and this provides an excellent example of the use of our techniques. The work presents theorems concerning the multivariate normal distribution and presents the first-order conditions and the information matrix. It then considers a system of non-linear regression equations with normal errors.

Falani*et al.* (2020) in the beginner's guide to maximum likelihood estimation discussed the basic theory of maximum likelihood, the advantages and disadvantages of maximum likelihood estimation, the log-likelihood function, the modeling application, and the conditional maximum likelihood function. In addition, a simple application of maximum likelihood estimation to a linear regression model. Thamer and Raoudha (2021) studied Lindley distribution with three parameters because of its high flexibility in modeling lifetime datasets. The parameters were estimated by five methods namely maximum likelihood estimation (MLE), ordinary least square (OLS), weighted mean squares, maximum product of spacing, and Cramer von Mises. Simulation experiments were performed with different sample sizes and different parameter values. The different methods were compared on the generated data by mean square error and mean absolute error.

Helu (2022) estimated the shape parameters of the Kumaraswamy distribution by utilizing the maximum product spacing (MPS) method. The asymptotic normality properties of the estimators are implemented to obtain confidence intervals. In addition, bootstrap confidence intervals are calculated. Monte Carlo simulations have been carried out to compare the maximum product spacing (MPS) and the maximum likelihood estimation (MLE) methods. To assess the effectiveness of the proposed procedure, a numerical example based on real data is presented. Singh *et al.,* (2014) studied the consequences of using maximum product spacing as an alternative to maximum likelihood estimation. The problem of point estimation of the parameter of exponential distribution was considered. The proposed estimates have been compared with those based on the maximum likelihood based on simulated samples from an exponential distribution.

Nassar and Farouq (2022) used the method of maximum product of spacing estimation to evaluate and estimate the efficiency of a new distribution called the Kies exponential distribution. Extensive simulation studies were carried out. Based on the simulation outcomes and real data analysis, maximum product spacing was recommended to evaluate and estimate the parameters of the Kies exponential distribution. Volovskiy and Udo (2020) studied the connections between the method of maximum product of spacing estimation and the method of maximum likelihood estimation using exponential distribution and Pareto distribution. The maximum product of the spacing predictor turns out to be useful to predict more than the maximum likelihood estimation predictor. A real data set was analyzed.

RESEARCH METHODOLOGY

According to Bourguignon *et al.*, (2014), a CDF and pdf of the odd exponential-G family of distributions are given by:

$$F_{OEG}\left(x;\delta,\xi\right)=1-\exp\left\{-\frac{\delta M\left(x;\xi\right)}{1-M\left(x;\xi\right)}\right\};\ \forall x;\delta,\xi>0 \qquad (1)$$

$$f_{OEG}\left(x;\delta,\xi\right)=\frac{\delta m\left(x;\xi\right)}{\left(1-M\left(x;\xi\right)\right)^2}\exp\left\{-\frac{\delta M\left(x;\xi\right)}{1-M\left(x;\xi\right)}\right\};\ \forall x;\delta,\xi>0 \qquad (2)$$

where $\delta>0$ is the scale parameter and $x>0$, $m\left(x;\xi\right)$ *and* $M\left(x;\xi\right)$ are the pdf and CDF and ξ is the parameters' vector of the baseline distribution.

According to Sadiq *et al.*, (2023), a random variable X is said to have Weibull distribution with scale parameter α and shape parameter β if its pdf and CDF are given as,

$$m\left(x;\alpha,\beta\right)=\beta\alpha^{-\beta}x^{\beta-1}\exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\};\ x,\alpha,\beta>0 \qquad (3)$$

$$M\left(x;\alpha,\beta\right)=1-\exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\};\ x,\alpha,\beta>0 \qquad (4)$$

The Odd Exponential-Weibull Distribution

Suppose that the baseline distribution M has Weibull distribution with pdf and cdf as in equations (3) and (4), then the pdf and cdf of Odd Exponential-Weibull distribution (OE-W) are defined by inserting equations (3) and (4) in equation (1) and equation (2) respectively.

$$f_{OEW}\left(x;\delta,\alpha,\beta\right)=\frac{\delta\left(\beta\alpha^{-\beta}x^{\beta-1}\exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\}\right)}{\left(\exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\}\right)^2}\exp\left\{-\frac{\delta\left(1-\exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\}\right)}{\exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\}}\right\};\ \forall x;\delta,\alpha,\beta>0 \qquad (5)$$

$$F_{OEW}(x;\delta,\alpha,\beta)=1-\exp\left\{-\frac{\delta\left(1-\exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\}\right)}{\exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\}}\right\};\ \forall x;\delta,\alpha,\beta>0 \quad (6)$$

The PDF and CDF plots of the Odd Exponential-Weibull Distribution keeping one parameter constant and varying others are presented in Figures 1 and 2 respectively.

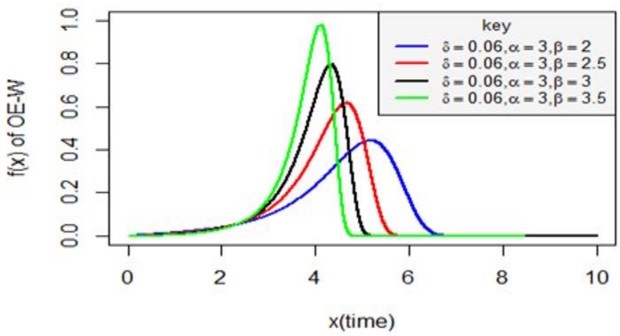

FIGURE 1: PDF Plot of Odd Exponential-Weibull Distribution.

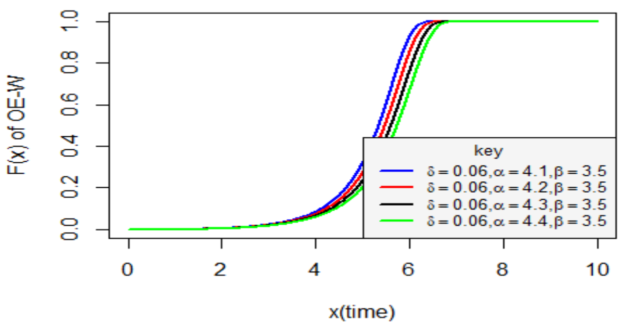

FIGURE 2: CDF Plot of Odd Exponential-Weibull Distribution.

Survival and Hazard Functions of OE-W Distribution

The survival function of a random variable X which follows the OE-W distribution is given by

$$S_{OEW}(x;\delta,\alpha,\beta)=\exp\left\{-\frac{\delta\left(1-\exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\}\right)}{\exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\}}\right\} \quad (7)$$

The Survival function plots of the Odd Exponential-Weibull Distribution keeping one parameter constant and varying others are presented in Figure 3.

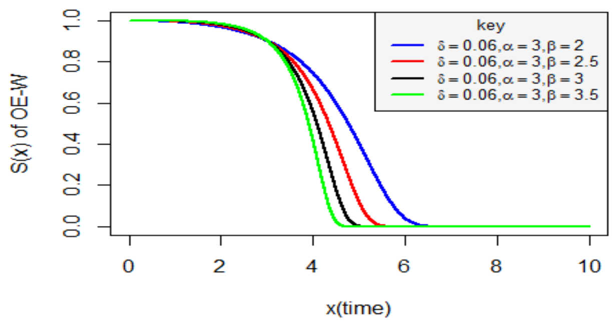

FIGURE 3: Survival Function Plot of the Odd Exponential-Weibull Distribution.

The hazard function of a random variable X which follows the OE-Weibull distribution is given by

$$h_{OEW}(x;\delta,\alpha,\beta)=\frac{\delta\beta\alpha^{-\beta}x^{\beta-1}}{\exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\}} \quad (8)$$

The hazard function plots of the Odd Exponential-Weibull Distribution keeping one parameter constant and varying others are presented in Figure 4.

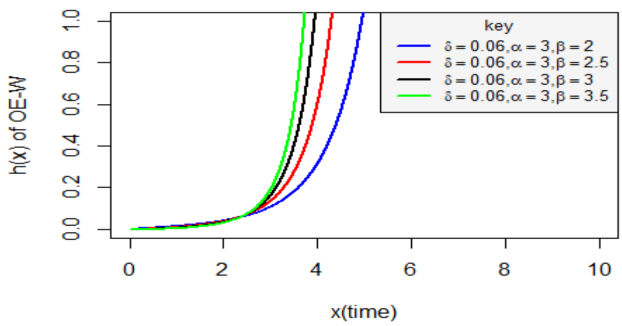

FIGURE 4: Hazard Function Plot of the Odd Exponential-Weibull Distribution.

RESULTS AND DISCUSSIONS

Application to Real-Life Datasets

At this point, we used some of the existing datasets to compare the performance of the proposed model and other related distributions. The PDF and CDF of competing distributions are the Odd Frechet-Odd-Weibull distribution developed by Ul-Haq and Elgarhy (2018); the Extended Weibull distribution developed by Xie *et al.* (2002); the Weibull-Weibull distribution developed by Bourguinon *et al.* (2014); and the New Weibull-Weibull distribution developed by Tahir *et al.* (2014) are respectively given as;

$$f_{OFW}(x;\delta,\alpha,\beta)=\frac{\delta\left(\beta\alpha^{-\beta}x^{\beta-1}\exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\}\right)}{\left(\exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\}\right)^{\delta+1}}\left(\exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\}\right)^{\delta-1}\exp\left\{-\left(\frac{\exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\}}{1-\exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\}}\right)^{\delta}\right\} \quad (9)$$

$$F_{OFW}\left(x;\delta,\alpha,\beta\right)=\exp\left\{-\left(\frac{\exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\}}{1-\exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\}}\right)^{\delta}\right\} \tag{10}$$

$$f_{EW}\left(x;\delta,\alpha,\beta\right)=\delta\beta\left(\frac{x}{\alpha}\right)^{\beta-1}\exp\left\{\left(\frac{x}{\alpha}\right)^{\beta}+\delta\alpha\left(1-\exp\left\{\left(\frac{x}{\alpha}\right)^{\beta}\right\}\right)\right\} \tag{11}$$

$$F_{EW}\left(x;\delta,\alpha,\beta\right)=1-\exp\left\{\delta\alpha\left(1-\exp\left\{\left(\frac{x}{\alpha}\right)^{\beta}\right\}\right)\right\} \tag{12}$$

$$f_{WW}\left(x;\delta,\lambda,\alpha,\beta\right)=\delta\lambda\beta\alpha^{-\beta}x^{\beta-1}\exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\}\frac{\left(1-\exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\}\right)^{\lambda-1}}{\left(\exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\}\right)^{\lambda+1}}\exp\left\{-\delta\left(\frac{1-\exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\}}{\exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\}}\right)^{\lambda}\right\} \tag{13}$$

$$F_{WW}\left(x;\delta,\lambda,\alpha,\beta\right)=1-\exp\left\{-\delta\left(\frac{1-\exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\}}{\exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\}}\right)^{\lambda}\right\} \tag{14}$$

$$f_{NWW}\left(x;\delta,\lambda,\alpha,\beta\right)=\delta\lambda\frac{\beta\alpha^{-\beta}x^{\beta-1}\exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\}}{\left(\exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\}\right)}\left(-\log\left(\exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\}\right)\right)^{\lambda-1}\exp\left\{-\delta\left(-\log\left(\exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\}\right)\right)^{\lambda}\right\} \tag{15}$$

$$F_{NWW}\left(x;\delta,\lambda,\alpha,\beta\right)=1-\exp\left\{-\delta\left(-\log\left(\exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\}\right)\right)^{\lambda}\right\} \tag{16}$$

Application to Survival Times (in months) of AAML Dataset

The data set analysed by Yakubu and Doguwa (2017) represents Survival times (in months) of a sample of 101 patients with Advanced Acute Myelogenous Leukaemia (AAML). The data are: 0.03, 8.882, 41.118, 6.151, 17.303, 0.493, 9.145, 45.033, 6.217, 17.664, 0.855, 11.48, 46.053, 6.447, 18.092, 1.184, 11.513, 46.941, 8.651, 18.092, 1.283, 12.105, 48.289, 8.717, 18.750, 1.48, 12.796 ,57.401, 9.441, 20.625, 1.776, 12.993, 58.322, 10.329, 23.158, 2.138, 13.849, 60.625, 11.48, 27.73, 2.5, 16.612, 0.658, 12.007, 31.184, 2.763, 17.138, 0.822, 12.007, 32.434, 2.993, 20.066, 1.414, 12.237, 35.921, 3.224, 20.329, 2.5, 12.401, 42.237, 3.421, 22.368, 3.322, 13.059, 44.638, 4.178, 26.776, 3.816, 14.474, 46.48, 4.441, 28.717, 4.737, 15, 47.467, 5.691, 28.717, 4.836, 15.461, 48.322, 5.855, 32.928, 4.934, 15.757, 56.086, 6.941, 33.783, 5.033, 16.48, 6.941, 34.211, 5.757, 16.711, 7.993, 34.77, 5.855, 17.204, 8.882, 39.539, 5.987, 17.237.

TABLE 1: M.L.E Goodness of Fit Measures for AAML Dataset.

Model	Parameters	Estimates	Standard Error	Goodness of Fit Measures	Rank
	δ	3.762e-05	2.814e-05	LL (-391.3732)	
	α	1.514e-01	9.758e-03	AIC (788.7464)	
OE-W	β	6.980e-04	2.462e-04	CAIC (788.9913)	1
				BIC (796.6213)	
				HQIC (791.9352)	
	δ	1.05885	0.23465	LL (-415.4612)	
	α	0.36425	0.03202	AIC (836.9224)	
OFW	β	0.44900	0.10506	CAIC (837.1673)	5
				BIC (844.7973)	
				HQIC (840.1112)	

Model	Parameters	Estimates	Standard Error	Goodness of Fit Measures	Rank
EW	δ	1.101072	0.222415	LL (-395.2038)	2
	α	0.005601	0.000274	AIC (796.4076)	
	β	0.195931	0.004752	CAIC (796.6525)	
				BIC (804.2825)	
				HQIC (799.5964)	
WW	δ	1.040e-01	2.240e-02	LL (-406.5038)	4
	λ	1.317e-01	9.367e-03	AIC (821.0076)	
	α	1.417e-01	2.666e-05	CAIC (821.42)	
	β	5.588e-01	5.299e-05	BIC (831.5075)	
				HQIC (825.2594)	
NWW	δ	1.659e-01	2.204e-02	LL (-402.7362)	3
	λ	6.871e-01	4.987e-02	AIC (813.4724)	
	α	2.372e+00	2.015e-07	CAIC (813.8848)	
	β	1.109e+00	3.624e-06	BIC (823.9723)	
				HQIC (817.7242)	

Source: Fieldsurvey, 2024.

The results presented in Table 1 reflect the comparative analysis of some statistical models fitted to the AAML dataset, with an emphasis on their goodness-of-fit measures. The table provides parameters and their estimates for each model, along with standard errors and various goodness-of-fit criteria such as Log-Likelihood (LL), Akaike Information Criterion (AIC), Consistent Akaike Information Criterion (CAIC), Bayesian Information Criterion (BIC), and Hannan-Quinn Information Criterion (HQIC). The log-likelihood (LL) values indicate the likelihood of the data given the model parameters. Among the models, the OEW model has the highest LL (-391.3732), suggesting it provides the best fit to the data by maximizing the likelihood. The AIC values assess the trade-off between the goodness of fit and the complexity of the model, where lower values indicate a better model. The OEW model exhibits the lowest AIC (788.7464), reinforcing its superior performance in balancing fit and complexity. The EW model also performs well with an AIC of 796.4076, ranking second among the models. The Consistent Akaike Information Criterion (CAIC), Bayesian Information Criterion (BIC), and Hannan-Quinn Information Criterion (HQIC): Similar to AIC, these criteria penalize complexity models, with the CAIC and BIC placing more emphasis on model simplicity.

The OEW model consistently outperforms the others across these measures, with the lowest CAIC (788.9913), BIC (796.6213), and HQIC (791.9352) values, highlighting its robustness as the best-fitting model. The EW model also ranks second in these metrics, indicating it is a strong alternative, particularly when model simplicity is prioritized. The model ranking, based on the overall goodness-of-fit measures, the OEW model is ranked first, indicating it is the most suitable model for the dataset among those considered.

This is followed by the EW model in second place. Other models such as NWW, WW, and OFW exhibit higher AIC, BIC, and HQIC values, which indicate less optimal fits compared to OEW and EW. The analysis demonstrates that the OEW model maximizes the goodness of fit for the AAML dataset, as evidenced by its superior performance across multiple criteria including LL, AIC, CAIC, BIC, and HQIC. The EW model also shows a competitive performance, ranking second. The selection of the OEW model is supported by its ability to provide a robust fit while maintaining a balance between model complexity and predictive accuracy.

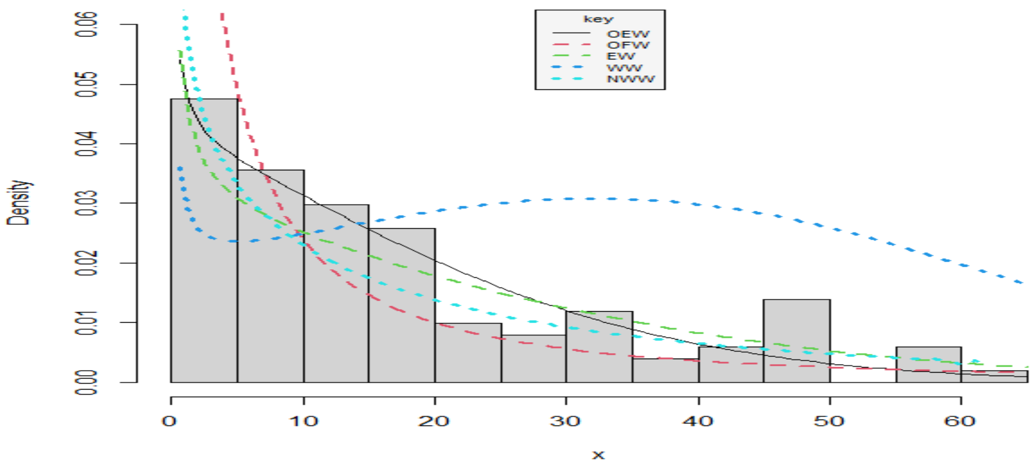

FIGURE 5: Histogram Plot of Survival Times (in months) of AAML Dataset.

Figure 5 represents a histogram and density plots that visualize the distribution of survival times (in months) for a cohort of 101 patients with Advanced Acute Myelogenous Leukemia (AAML). The curves represent different extensions of the Weibull model, such as OEW, OFW, EW, WW, and NWW. The histogram reveals a right-skewed distribution of survival times, indicating that a majority of patients experienced shorter survival periods, with a tail extending towards longer survival times. This pattern is commonly observed in survival data and suggests the need for survival analysis models that can accommodate such right-skewed distributions. The survival times of AAML patients exhibit a characteristic right-skewed distribution, as depicted in the histogram. The overlaid density curves representing different Weibull model extensions offer potential approaches for modelling and understanding the survival experience of this patient population.

TABLE 2: M.L.E Goodness of Fit Measures for Remission Time of Bladder Cancer Dataset.

Model	Parameters	Estimates	Standard Error	Goodness of Fit Measures	Rank
	δ	5.805e-05	1.202e-05	LL (-422.521)	
	α	1.559e-01	7.536e-04	AIC (851.042)	
OE-W	β	1.412e-03	3.140e-04	CAIC (851.2355)	1
				BIC (859.5981)	
				HQIC (854.5184)	
	δ	1.40049	0.16828	LL (-428.1673)	
	α	0.42681	0.02324	AIC (862.3346)	
OFW	β	0.40994	0.06421	CAIC (862.5281)	2
				BIC (870.8907)	
				HQIC (865.811)	
	δ	1.527686	0.223849	LL (-437.4101)	
	α	0.010282	0.001148	AIC (880.8202)	
EW	β	0.198029	0.004389	CAIC (881.0137)	3
				BIC (889.3763)	
				HQIC (884.2966)	
	δ	0.748089	0.184193	LL (-623.537)	
	λ	0.192392	0.027448	AIC (1255.074)	
WW	α	0.002728	0.000159	CAIC (1255.399)	5
	β	0.179744	0.018205	BIC (1266.482)	
				HQIC (1259.709)	
	δ	0.044195	0.004041	LL (-527.7562)	
NWW	λ	1.002325	0.068178	AIC (1063.512)	4
	α	2.464115	0.002538	CAIC (1063.838)	
	β	1.044347	0.082934	BIC (1074.921)	
				HQIC (1068.148)	

Source: Field Survey, 2024

Application to Remission Time of Bladder Cancer Dataset

The data set was reported by Oguntunde *et al.,* (2016) which represents the remission time of a random sample of 128 bladder cancer patients. The data are: 0.08, 2.09, 3.48,4.87, 6.94, 8.66, 13.11, 23.63, 0.20, 2.23, 3.52, 4.98, 6.97, 9.02, 13.29, 0.40, 2.26, 3.57,5.06, 7.09, 9.22, 13.80, 25.74, 0.50, 2.46, 3.64, 5.09, 7.26, 9.47, 14.24, 25.82, 0.51, 2.54,3.70, 5.17, 7.28, 9.74, 14.76, 26.31, 0.81, 2.62, 3.82, 5.32, 7.32, 10.06, 14.77, 32.15, 2.64,3.88, 5.32, 7.39, 10.34, 14.83, 34.26, 0.90, 2.69, 4.18, 5.34, 7.59, 10.66, 15.96, 36.66, 1.05,2.69, 4.23, 5.41, 7.62, 10.75, 16.62, 43.01, 1.19, 2.75, 4.26, 5.41, 7.63, 17.12, 46.12, 1.26,2.83, 4.33, 5.49, 7.66, 11.25, 17.14, 79.05, 1.35, 2.87, 5.62, 7.87, 11.64, 17.36, 1.40, 3.02,4.34, 5.71, 7.93, 11.79, 18.10, 1.46, 4.40, 5.85, 8.26, 11.98, 19.13, 1.76, 3.25, 4.50, 6.25,8.37, 12.02, 2.02, 3.31, 4.51, 6.54, 8.5, 12.03, 20.28, 2.02, 3.36, 6.76, 12.07, 21.73, 2.07,3.36, 6.93, 8.65, 12.63, 22.69.

In Table 2, we evaluated the performance of five different survival models (OE-W, OFW, EW, WW, NWW) on the Remission Time of Bladder Cancer dataset. The assessment focused on the Maximum Likelihood Estimates (MLE) of the model parameters and a set of goodness-of-fit metrics, including the Log-Likelihood (LL), Akaike Information Criterion (AIC), Consistent AIC (CAIC), Bayesian Information Criterion (BIC), and Hannan-Quinn Information Criterion (HQIC). The MLEs represent the parameter values (δ, α, β) that maximize the likelihood function, indicating the most probable values given the observed data. Alongside these estimates, the standard errors are provided, offering insight into the precision of these estimates. For the OEW Model, the parameters (δ = 5.805e-05, α = 1.559e-01, β = 1.412e-03) were estimated with relatively low standard errors, indicating high precision. For the OFW Model, the estimates (δ = 1.40049, α = 0.42681, β = 0.40994) show higher values with moderate standard errors, suggesting reasonable precision but less certainty compared to the OE-W model. For the EW Model, the parameter estimates (δ = 1.527686, α = 0.010282, β = 0.198029) reveal a more modest fit with slightly larger standard errors.

The WW and NWW Models, these models exhibited higher standard errors, particularly the WW model (with δ = 0.748089, λ = 0.192392, α = 0.002728 β = 0.179744), suggesting lower precision in these parameter estimates. The goodness-of-fit measures provide a quantitative assessment of how well each model describes the observed data. Lower values in these metrics generally indicate better-fitting models. For Log-Likelihood (LL), the OE-W model yielded the highest LL (-422.521), suggesting it provides the best fit among the models. The NWW and WW models had the lowest LL values (-527.7562 and -623.537, respectively), indicating poorer fits. For the Akaike Information Criterion (AIC), the OE-W model again performed best with the lowest AIC (851.042), reflecting a good balance between model fit and complexity. The WW model showed the highest AIC (1255.074), reinforcing its inadequacy in fitting the data well. These metrics follow a similar pattern, with the OE-W model consistently ranking as the best fit (CAIC = 851.2355, BIC = 859.5981, HQIC = 854.5184), while the WW model ranks the lowest (CAIC = 1255.399, BIC = 1266.482, HQIC = 1259.709). The OEW Model has the lowest values across LL, AIC, CAIC, BIC, and HQIC, the OE-W model is the best-fitting model for the Remission Time of Bladder Cancer dataset. Its estimates are precise, and it provides the most accurate representation of the data.

The OFW Model ranks second, showing a reasonable fit, but it is less optimal than the OE-W model. The EW Model, while performing better than the WW and NWW models, the EW model is outperformed by both the OE-W and OFW models. The WW and NWW Models are ranked lowest, indicating poor fit and less reliability in parameter estimation for this dataset. The analysis emphasizes the superiority of the OEW model for modelling the remission times in bladder cancer, given its superior fit and precise parameter estimates. The use of multiple goodness-of-fit measures provides a robust framework for model comparison, highlighting the importance of considering both model fit and complexity in selecting the most appropriate model for survival data analysis. The results suggest that the OEW model is well-fitted for this dataset, offering a nuanced and accurate understanding of the remission time distribution in bladder cancer patients.

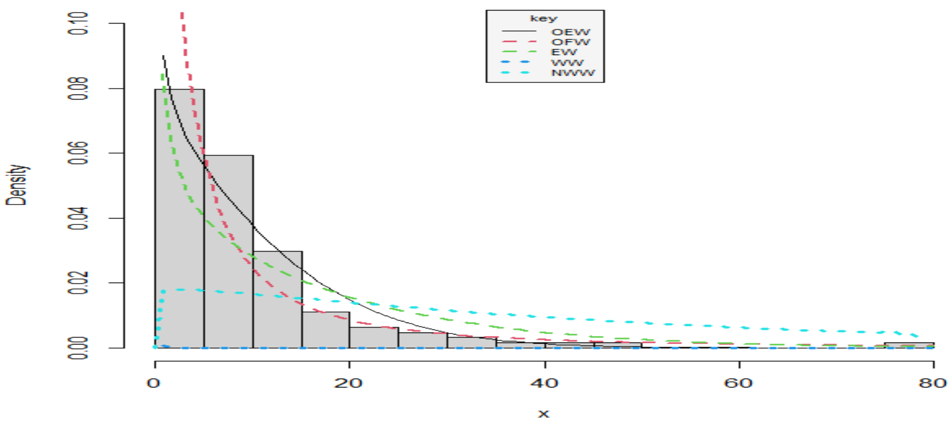

FIGURE 6: Histogram Plot of Remission Time of Bladder Cancer Dataset.

Figure 6 represents a histogram and density plot that visualize the distribution of the remission time of the bladder cancer dataset. The curves represent different extensions of the Weibull model, such as OEW, OFW, EW, WW, and NWW. The histogram of the density plots reveals a right-skewed distribution of remission times, indicating that a majority of patients experienced shorter remission periods, with a tail extending towards longer remission times.

Application to Strengths of Glass Fibres Dataset

The data set analysed by Lakshmi et al., (2020) represents the strengths of 3.5 cm glass fibres from 62 observations obtained by the National Physical Laboratory. The data are: 4.99, 3.97, 2.18, 3.14, 2.19, 4.96, 2.66, 4.98, 3.37, 2.85, 4.88, 3.27, 4.29,3.29, 4.10, 4.76, 4.49, 4.24, 2.85, 3.16, 2.16, 2.34, 3.84, 4.52, 2.89, 4.87, 2.87, 2.40, 4.30, 3.73, 3.45, 4.98, 4.43, 2.09, 2.30, 2.89, 2.53, 2.01, 4.94, 2.23, 4.15, 2.73, 3.59, 3.27, 4.70, 2.14, 4.84, 4.46, 4.42, 2.57, 3.64, 3.54, 3.70, 3.95, 2.98, 4.23, 3.78, 4.84, 3.54, 3.03, 2.98, 3.89.

TABLE 3: M.L.E Goodness of Fit Measures for Strengths of Glass Fibres Dataset.

Model	Parameters	Estimates	Standard Error	Goodness of Fit Measures	Rank
OE-W	δ α β	2.032e-02 4.433e-01 3.087e-05	9.822e-03 5.711e-02 1.907e-05	LL (-82.88169) AIC (171.7634) CAIC (172.1772) BIC (178.1448) HQIC (174.2689)	1
OFW	δ α β	16.386555 0.670702 0.037742	0.353701 0.001410 0.004905	LL (-105.5214) AIC (217.0428) CAIC (217.4566) BIC (223.4242) HQIC (219.5483)	3
EW	δ α β	2.21011 0.03271 0.25541	0.63114 0.02137 0.01342	LL (-165.5327) AIC (337.0654) CAIC (337.4792) BIC (343.4468) HQIC (339.5709)	4
WW	δ λ α β	1.216e+00 9.721e-03 8.388e-01 2.030e+00	1.615e-01 1.245e-03 3.691e-05 5.695e-05	LL (-211.4376) AIC (430.8752) CAIC (431.577) BIC (439.3837) HQIC (434.2159)	5
NWW	δ λ α β	3.416e-03 1.809e+00 3.777e-01 1.332e+00	1.017e-03 1.002e-01 6.516e-06 1.467e-05	LL (-96.86083) AIC (201.7217) CAIC (202.4234) BIC (210.7217) HQIC (205.0623)	2

Source: Field Survey, 2024

The results presented in Table 3 reflect the comparative analysis of several statistical models fitted to the strengths of glass fibres, with an emphasis on their goodness-of-fit measures. The table provides parameter estimates for each model, along with standard errors and various goodness-of-fit criteria such as Log-Likelihood (LL), Akaike Information Criterion (AIC), Consistent Akaike Information Criterion (CAIC), Bayesian Information Criterion (BIC), and Hannan-Quinn Information Criterion (HQIC). The log-likelihood (LL) values indicate the likelihood of the data given the model parameters. Among the models, the OEW model has the highest LL (-82.88169), suggesting it provides the best fit to the data by maximizing the likelihood. The AIC values assess the trade-off between the goodness of fit and the complexity of the model, where lower values indicate a better model. The OEW model exhibits the lowest AIC (171.7634), reinforcing its superior performance in balancing fit and complexity. The NWW model also performs well with an AIC of 201.7217, ranking second among the models.

The Consistent Akaike Information Criterion (CAIC), Bayesian Information Criterion (BIC), and Hannan-Quinn Information Criterion (HQIC): Similar to AIC, these criteria penalize complexity models, with the CAIC and BIC placing more emphasis on model simplicity. The OEW model consistently outperforms the others across these measures, with the lowest CAIC (172.1772), BIC (178.1448), and HQIC (174.2689) values, highlighting its robustness as the best-fitting model. The NWW model also ranks second in these metrics, indicating it is a strong alternative, particularly when model simplicity is prioritized.

The model ranking, based on the overall goodness-of-fit measures, the OEW model is ranked first, indicating it is the most suitable model for the dataset among those considered. This is followed by the NWW model in second place. Other models such as OFW, EW, and WW exhibit higher AIC, BIC, and HQIC values, which indicate less optimal fits compared to OEW and NWW.

The analysis demonstrates that the OEW model maximizes the goodness of fit for the AAML dataset, as evidenced by its superior performance across multiple criteria including LL, AIC, CAIC, BIC, and HQIC. The NWW model also shows a competitive performance, ranking second. The selection of the OEW model is supported by its ability to provide a robust fit while maintaining a balance between model complexity and predictive accuracy.

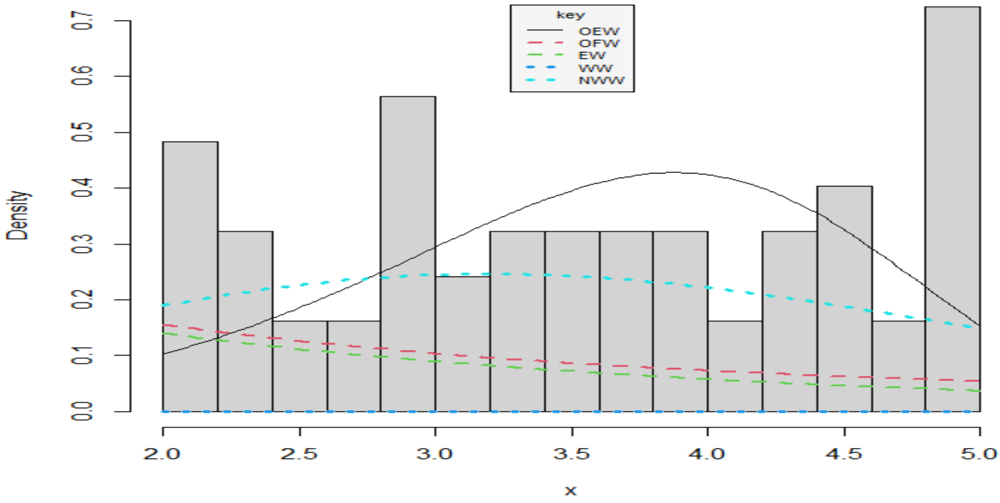

FIGURE 7: Histogram Plot of Strengths of Glass Fibres.

Figure 7 presents the histogram density plot of the strengths of the glass fibres dataset. The histogram of the density plot exhibits a right-skewed distribution, suggesting a potential fit with the extensions of the Weibull models. The density curves visually support this notion, with the OEW curve demonstrating a closer association with the data.

CONCLUSION

The study examines some of the existing parametric distributional models in accommodating various datasets and develops an extension termed the new odd exponential-Weibull distribution with applications to survival datasets. Building upon prior work, this new distribution enhanced flexibility across several datasets. Thus, the study introduced the new distribution and evaluated its performance using lifetime and survival datasets. In addition, the research established the potential of the developed new distribution and its variants as promising alternatives in modelling positive data.

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