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Generalization of the Lorentz Transformation of the Electromagnetic Four-potential and Concerns About Electrostatic Shielding

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ABSTRACT

This paper is about generalizing the formula for the Lorentz transformation of the electromagnetic four-potential. In the conventional Lorentz transformation method, the X, Y, or Z coordinate axis is determined as the direction of motion of system S' relative to system S. Then, a four-vector is transformed using a transformation matrix that matches the direction of motion of the system S' [1][2]. If the system S' moves in an arbitrary direction, it is possible to determine each component of the matrix by rotating the coordinate axes, but this process is complicated [3]. In this paper, a transformation formula is presented that allows the electromagnetic four-potential to be easily transformed into a Lorentz transform even if the direction of motion of the system S' is arbitrary. In the Discussion, new problems in electrostatic shielding were presented by discussing the motion of charged particles using the compact transformation equation.

Keywords: Lorentz transform; electromagnetic four-potential; constant potential field; electrostatic shielding.

INTRODUCTION

This research aims to improve the reliability of electrostatic shields. This paper has revealed new concerns about the weaknesses of electrostatic shields.

There is an electromagnetic four-potential $\left(\phi/c, A_x, A_y, A_z\right)$ on a system S, and the electromagnetic potential that appears in the system S' moving with a velocity $\mathbf{v}$ relative to the system S is $\left(\phi'/c, A'_x, A'_y, A'_z\right)$.

The Lorentz transformation of the electromagnetic four-potential that has been used conventionally refers only to the case where the system S' moves linearly in the directions of each of the three axes, and the following transformation formula is publicly known [4].

When the system S' is moving in the X-axis direction with a velocity v, the Lorentz transformation of the electromagnetic four-potential is

$$\begin{pmatrix} \phi'/c \\ A'_x \\ A'_y \\ A'_z \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \phi/c \\ A_x \\ A_y \\ A_z \end{pmatrix} \quad (1)$$

If the system S' is moving with velocity in the Y-axis direction,

$$\begin{pmatrix} \phi'/c \\ A'_x \\ A'_y \\ A'_z \end{pmatrix} = \begin{pmatrix} \gamma & 0 & -\gamma\beta & 0 \\ 0 & 1 & 0 & 0 \\ -\gamma\beta & 0 & \gamma & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \phi/c \\ A_x \\ A_y \\ A_z \end{pmatrix} \quad (2)$$

And if the system S' is moving with velocity in the axial Z direction,

$$\begin{pmatrix} \phi'/c \\ A'_x \\ A'_y \\ A'_z \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\beta & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} \phi/c \\ A_x \\ A_y \\ A_z \end{pmatrix} \quad (3)$$

Where

$$\gamma = 1/\sqrt{1 - v^2/c^2} \quad (4)$$

$$\beta = v/c \quad (5)$$

v is the velocity of motion in a system S'

c is light velocity

STEPS TO OBTAIN COMPACT FORMULAS

When equation (1) is written down for each component,

$$\begin{aligned} \phi' &= \gamma\phi - \gamma v_x A_x \\ A'_x &= -\gamma \frac{v_x \phi}{c^2} + \gamma A_x \\ A'_y &= A_y \\ A'_z &= A_z \end{aligned} \quad (6)$$

Similarly, the components of equations (2) and (3) can be written as

$$\begin{aligned} \phi' &= \gamma\phi - \gamma v_y A_y \\ A'_x &= A_x \\ A'_y &= -\gamma \frac{v_y \phi}{c^2} + \gamma A_y \\ A'_z &= A_z \end{aligned} \quad (7)$$

$$\begin{aligned} \phi' &= \gamma\phi - \gamma v_z A_z \\ A'_x &= A_x \\ A'_y &= A_y \\ A'_z &= -\gamma \frac{v_z \phi}{c^2} + \gamma A_z \end{aligned} \quad (8)$$

When the direction of the velocity of motion of the system S' is arbitrary, the electromagnetic potential seen from the system S' can be obtained by integrating and describing equations (6), (7), and (8), and the transformation equation is as follows.

$$\begin{aligned} \phi' &= \gamma\phi - \gamma \mathbf{v}\cdot\mathbf{A} \\ \mathbf{A}' &= -\gamma \frac{\phi}{c^2}\mathbf{v} + \gamma \mathbf{A}_{//} + \mathbf{A}_{\perp} \end{aligned} \quad (9)$$

Here, $\mathbf{A}_{//}$ and $\mathbf{A}_{\perp}$ are the parallel and perpendicular to $\mathbf{v}$, and can be defined as follows:

$$\begin{aligned} \mathbf{A}_{//} &= \frac{\mathbf{v}\cdot\mathbf{A}}{v^2}\mathbf{v} \\ \mathbf{A}_{\perp} &= \mathbf{A} - \mathbf{A}_{//} \\ &= \mathbf{A} - \frac{\mathbf{v}\cdot\mathbf{A}}{v^2}\mathbf{v} \end{aligned} \quad (10)$$

Substituting equation (10) into (9), we get

$$\begin{aligned} \phi' &= \gamma\phi - \gamma \mathbf{v}\cdot\mathbf{A} \\ \mathbf{A}' &= -\gamma \frac{\phi}{c^2}\mathbf{v} + (\gamma - 1)\frac{\mathbf{v}\cdot\mathbf{A}}{v^2}\mathbf{v} + \mathbf{A} \end{aligned} \quad (11)$$

This formula allows us to calculate the electromagnetic four-potentials ϕ' and $\mathbf{A}'$ as seen from a system S' moving with an arbitrary velocity vector $\mathbf{v}$.

VERIFICATION

The fact that equation (11) is reliable as a Lorentz transformation can be proven as follows. If the four-potential obeys the Lorentz transformation, the following relationship must hold.

$$\begin{aligned} &A'^2_x + A'^2_y + A'^2_z - \phi'^2 / c^2 \\ &= A_x^2 + A_y^2 + A_z^2 - \phi^2 / c^2 \end{aligned} \quad (12)$$

By substituting equation (11) into the left side of equation (12), the right side of equation (12) is obtained as follows:

$$\begin{aligned} &A'^2_x + A'^2_y + A'^2_z - \phi'^2 / c^2 \\ &= \gamma^2 \frac{1}{c^2}(\mathbf{v}\cdot\mathbf{A})^2 + \gamma^2 \frac{v^2}{c^2}\frac{\phi^2}{c^2} - \gamma^2 \frac{\phi^2}{c^2} - \gamma^2 \frac{1}{c^2}(\mathbf{v}\cdot\mathbf{A})^2 + A^2 \\ &= -\gamma^2 \frac{\phi^2}{c^2}\left(1 - \frac{v^2}{c^2}\right) + A^2 \\ &= -\frac{\phi^2}{c^2} + A^2 \\ &= A_x^2 + A_y^2 + A_z^2 - \frac{\phi^2}{c^2} \end{aligned} \quad (13)$$

In the process of this derivation, the following relationship was used:

$$\left(\gamma^2 - 1\right)\frac{1}{v^2} = \gamma^2 \frac{1}{c^2} \quad (14)$$

Therefore, it has been proven that equation (11) represents the Lorentz transformation of the electromagnetic four-potential.

ADVANTAGES OF EQUATION (11)

The conventional equations (1), (2), and (3) are written on the assumption that they deal with linear motion in the direction of the coordinate axes, but are not appropriate for curved motion.

The advantage of equation (11) obtained in this paper is that it can be applied even if the direction of the velocity vector is arbitrary, and it can also be easily applied when the direction of the velocity changes over time.

If we apply equation (11) under conditions where $v \ll c$, then since $\gamma \approx 1$, equation (11) can be simplified as follows:

$$\begin{aligned} \phi' &= \phi - \mathbf{v}\cdot\mathbf{A} \\ \mathbf{A}' &= -\frac{\phi}{c^2}\mathbf{v} + \mathbf{A} \end{aligned} \quad (15)$$

FIELD OF A SYSTEM ACCELERATING IN A CONSTANT POTENTIAL SPACE

There is a space in which the electromagnetic four-potential is given as follows:

$$\left(\phi, A_x, A_y, A_z\right) = \left(V, 0, 0, 0\right) \qquad (16)$$

Here, V is assumed to be a constant value regardless of location in space. The electromagnetic potential appearing in a system S' moving with a velocity $\mathbf{v}$ can be obtained by substituting equation (16) into (11) as follows:

$$\phi' = \gamma V$$
$$\mathbf{A}' = -\gamma \frac{V}{c^2} \mathbf{v} \qquad (17)$$

Moreover, the electric field $\mathbf{E}'$ and magnetic field $\mathbf{B}'$ appearing in the system S' are expressed by the following equations.

$$\mathbf{E}' = -\mathrm{grad}\ \phi' - \frac{\partial}{\partial t}\mathbf{A}' \qquad (18)$$

$$\mathbf{B}' = \mathrm{rot}\ \mathbf{A}' \qquad (19)$$

Substituting equation (17) into (18) and (19), we obtain the following equations.

$$\mathbf{E}' = \gamma \frac{V}{c^2} \frac{d\mathbf{v}}{dt} \qquad (20)$$

$$\mathbf{B}' = 0 \qquad (21)$$

Equation (20) means that in a constant potential space, an accelerating system generates an electric field proportional to the background potential V and the acceleration $d\mathbf{v}/dt$. Also, as shown in equation (21), there is no magnetic field.

If the electric field and magnetic field in the system S are Lorentz transformed into the electric field and magnetic field in a system S' without mediation the electromagnetic four-potential, equation (20) cannot be obtained. The reason is that some information is lost when the electromagnetic four-potential is converted into the electric field and magnetic field. The constant components contained in the electrostatic potential are lost via the operator grad. Also, the gradient components and constant vector components contained in the vector potential are lost via the operator rot.

EQUATION OF MOTION FOR A CHARGED PARTICLE MOVING IN A SPACE WITH A CONSTANT POTENTIAL

If there is a charged particle moving with the system S' and its charge is q, the force $\mathbf{F}$ acting on the particle can be described as follows:

$$\mathbf{F} = q\mathbf{E}' + \mathbf{F}_{ext} \qquad (22)$$

Here, $\mathbf{E}'$ is the electric field that appears in the system S', and $\mathbf{F}_{ext}$ is an external force acting on the charged particle due to a factor other than $\mathbf{E}'$.

Substituting equation (20) into equation (22), we get

$$\mathbf{F} = \gamma q \frac{V}{c^2} \frac{d\mathbf{v}}{dt} + \mathbf{F}_{ext} \qquad (23)$$

Here, the equation of motion of the particle is given by the following equation:

$$\mathbf{F} = m\frac{d\mathbf{v}}{dt} \qquad (24)$$

Where

m is the Mass of the particle

From equations (23) and (24), the following equation is obtained:

$$\mathbf{F}_{ext} = \left(m - \gamma q \frac{V}{c^2}\right)\frac{d\mathbf{v}}{dt} \qquad (25)$$

Since $m - \gamma q V / c^2$ on the right side of this equation is the dimension of mass, the particle has an apparent mass m' defined by the following equation.

$$m' = m - \gamma q V / c^2 \qquad (26)$$

This formula means that the apparent mass of a charged particle changes depending on the background potential V. This will affect the cyclotron frequency, for example.

In electronic devices that handle the movement of electrons very sensitively, if attention is not paid to the electric potential of the space in which the device is placed, the apparent mass of the electron will change, which can cause malfunctions.

Even when space probes have passed sufficient system tests before launch, numerous unexpected malfunctions have been reported when they are actually deployed in space. The main trigger for these malfunctions is thought to be electrical stimuli from the sun or the Earth's radiation belts [5].

The reliability of spacecraft electrostatic shielding is typically examined for effects of ESD, EMI, and EMP [6], but the above discussion suggests that the effects of external potentials should also be considered.

Even if the electric field inside the space inside the electrostatic shield of a probe is zero, the external electric potential will penetrate the inside of the shield, so even if there are electronic devices inside the shield, the electrons in the circuit will be affected by V according to equation (20).

To solve this problem, it is necessary to further improve the electrostatic shield of the probe.

CONCLUSIONS

The equations for the Lorentz transformation of the electromagnetic four-potential have been made compact, making it easier to apply these equations.

Even if the electric potential in a space is constant, a charged particle moving in that space may be affected by that electric potential. This prediction suggests that in an environment where high-voltage systems and electronic devices coexist, a Faraday shield installed to protect the electronic device may not function effectively.

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